Matematyczne modelowanie mózgu
(czyli o termodynamice)

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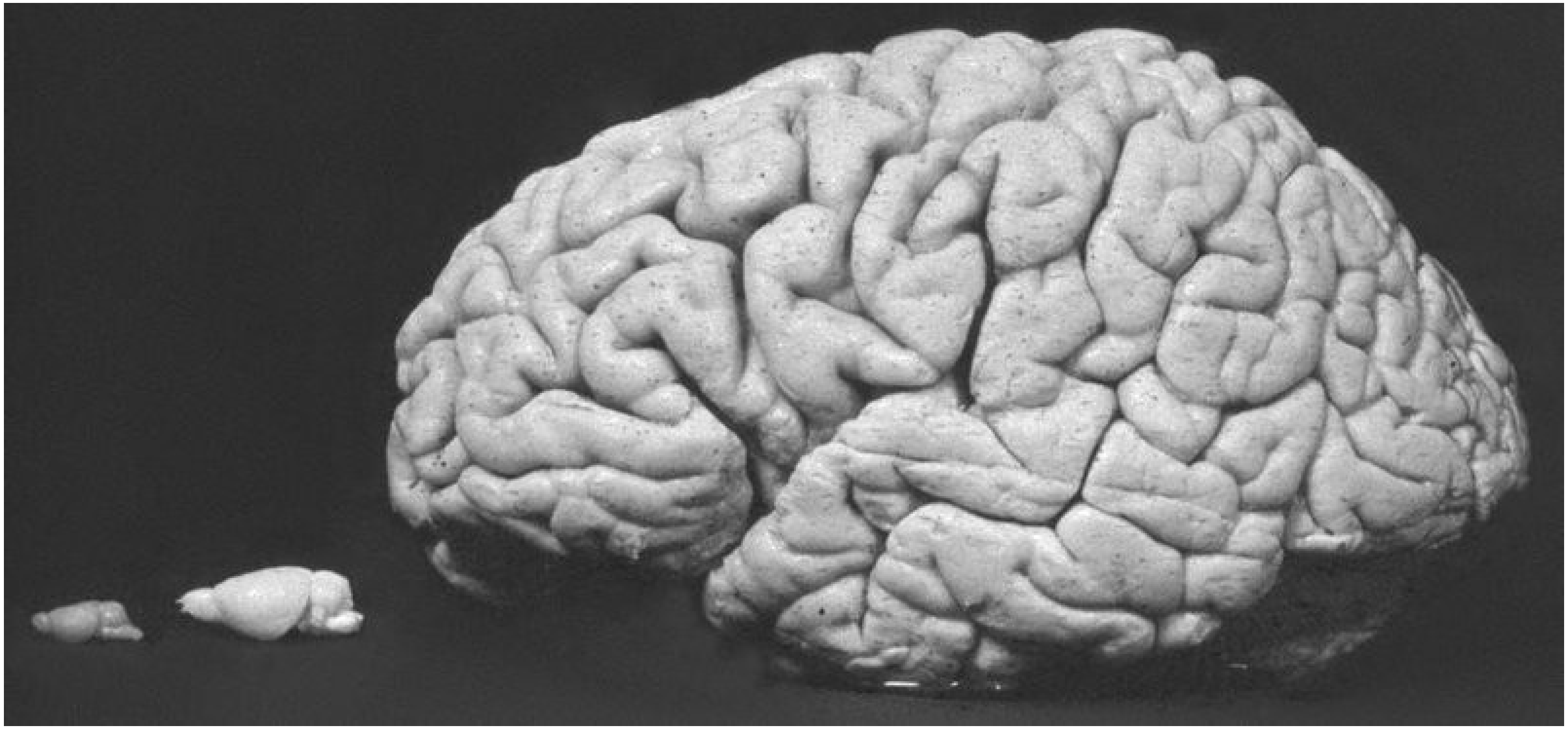
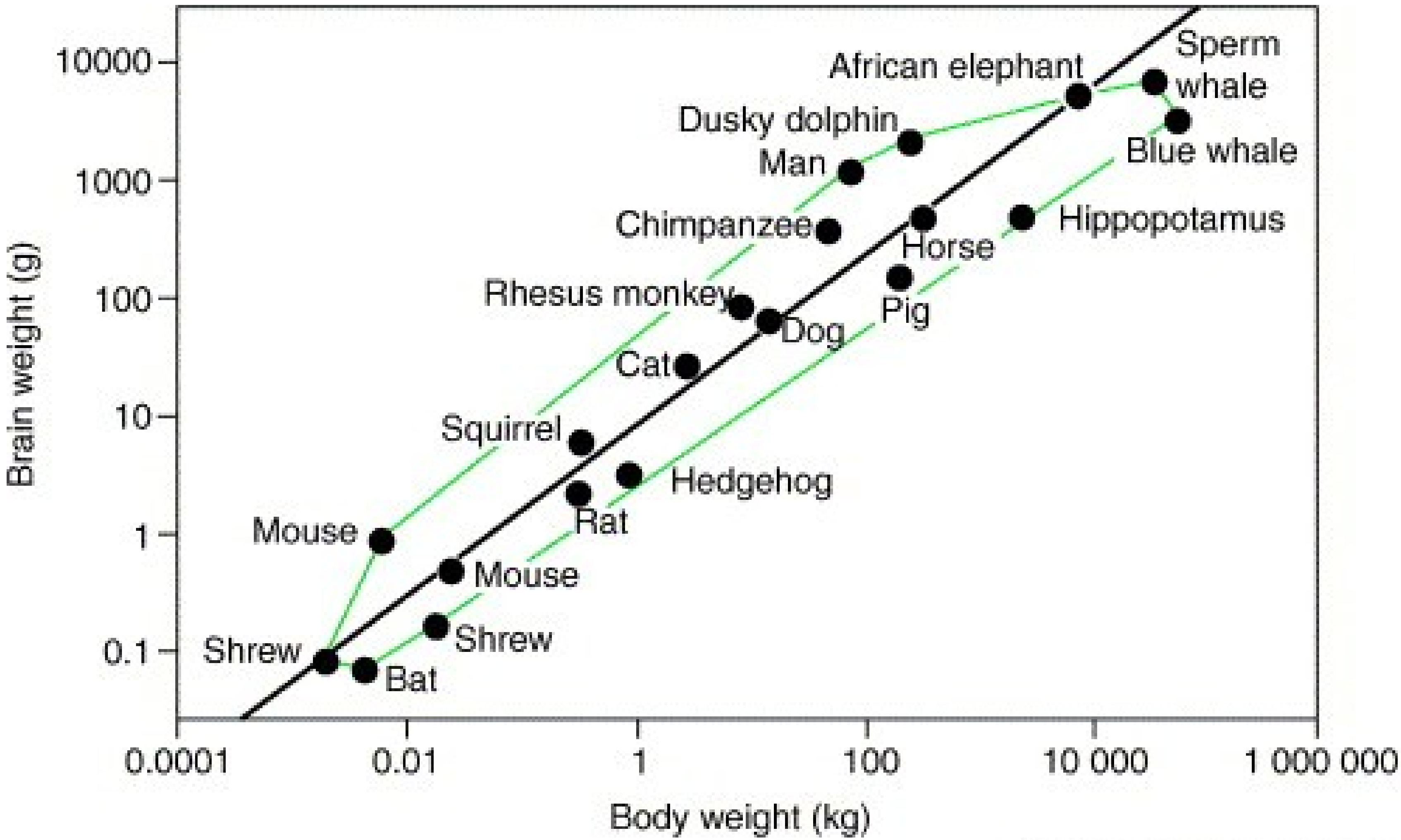
University of Warsaw

Brain sizes

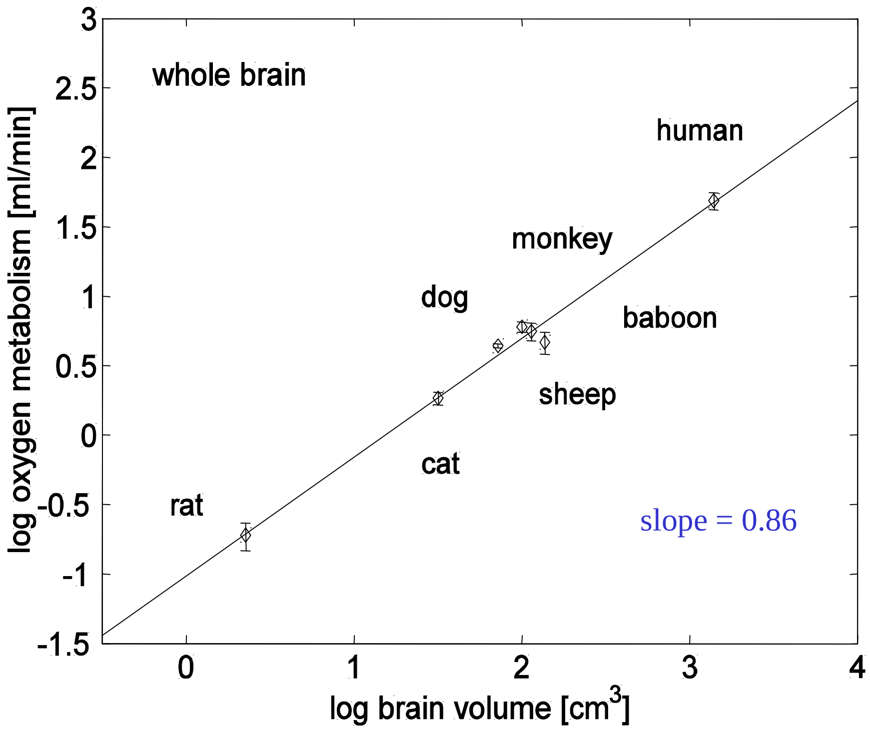


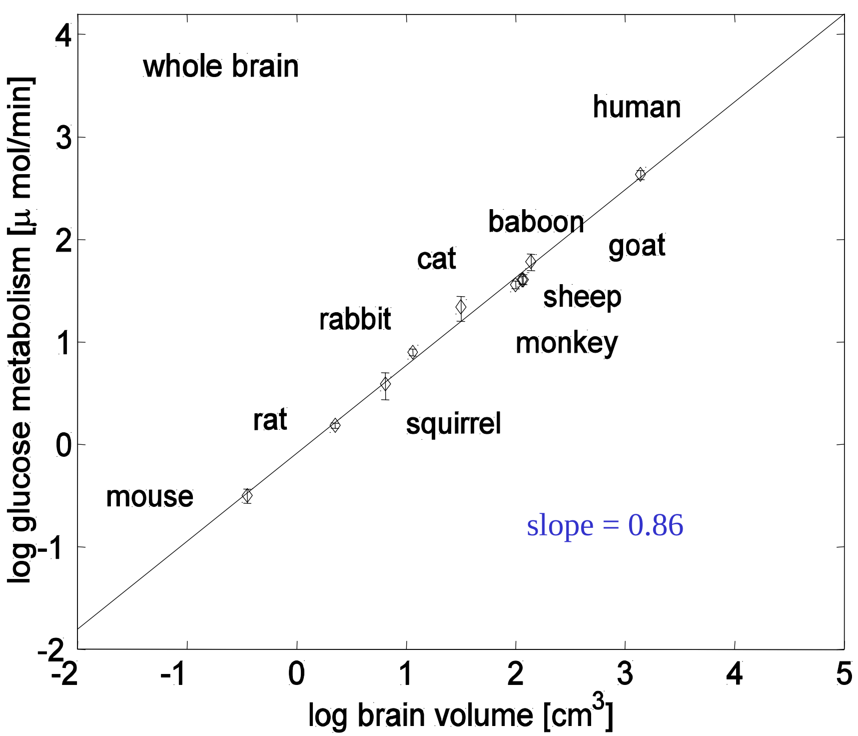
mouse rat human



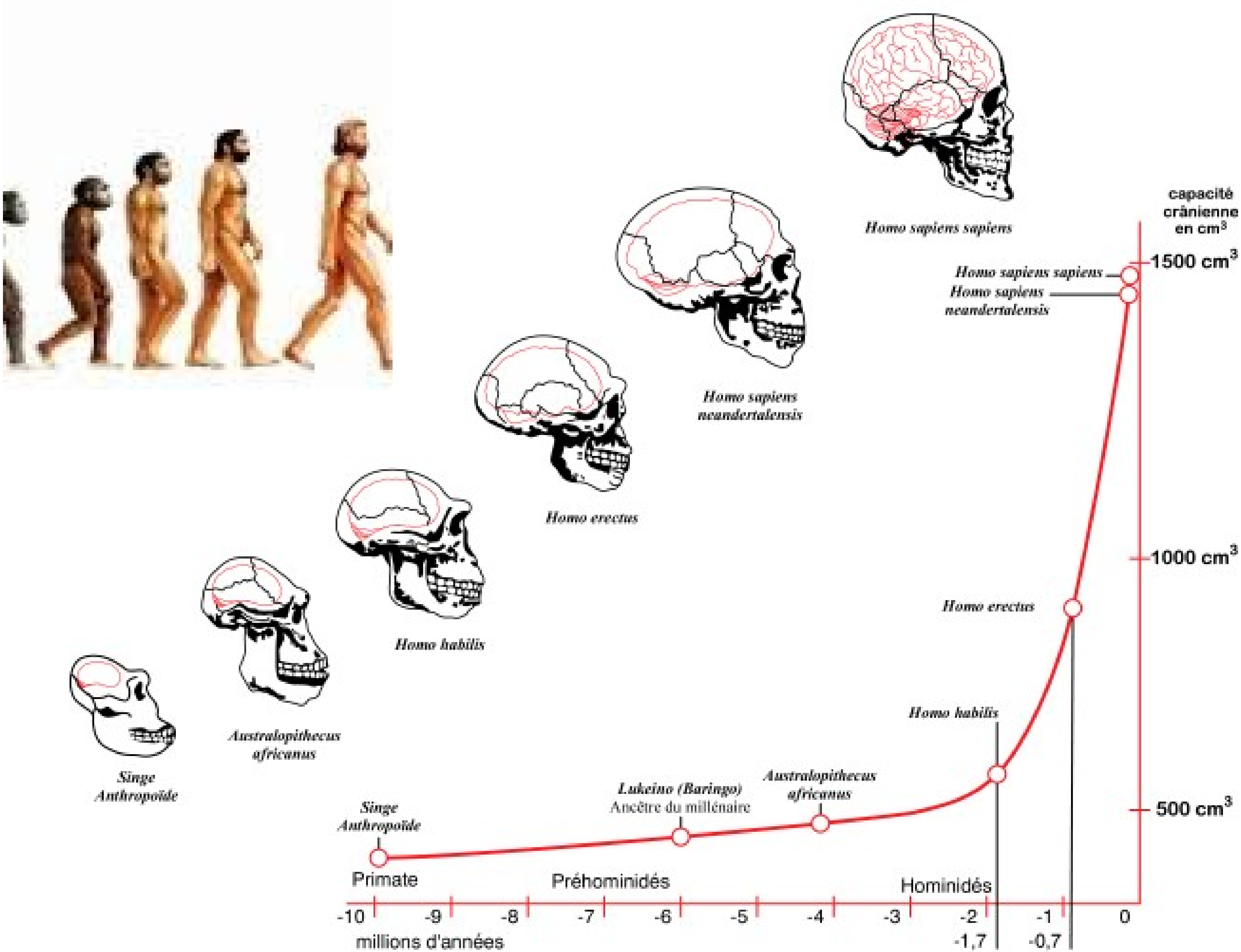
TRENDS in Cognitive Sciences

Global brain metabolic scaling





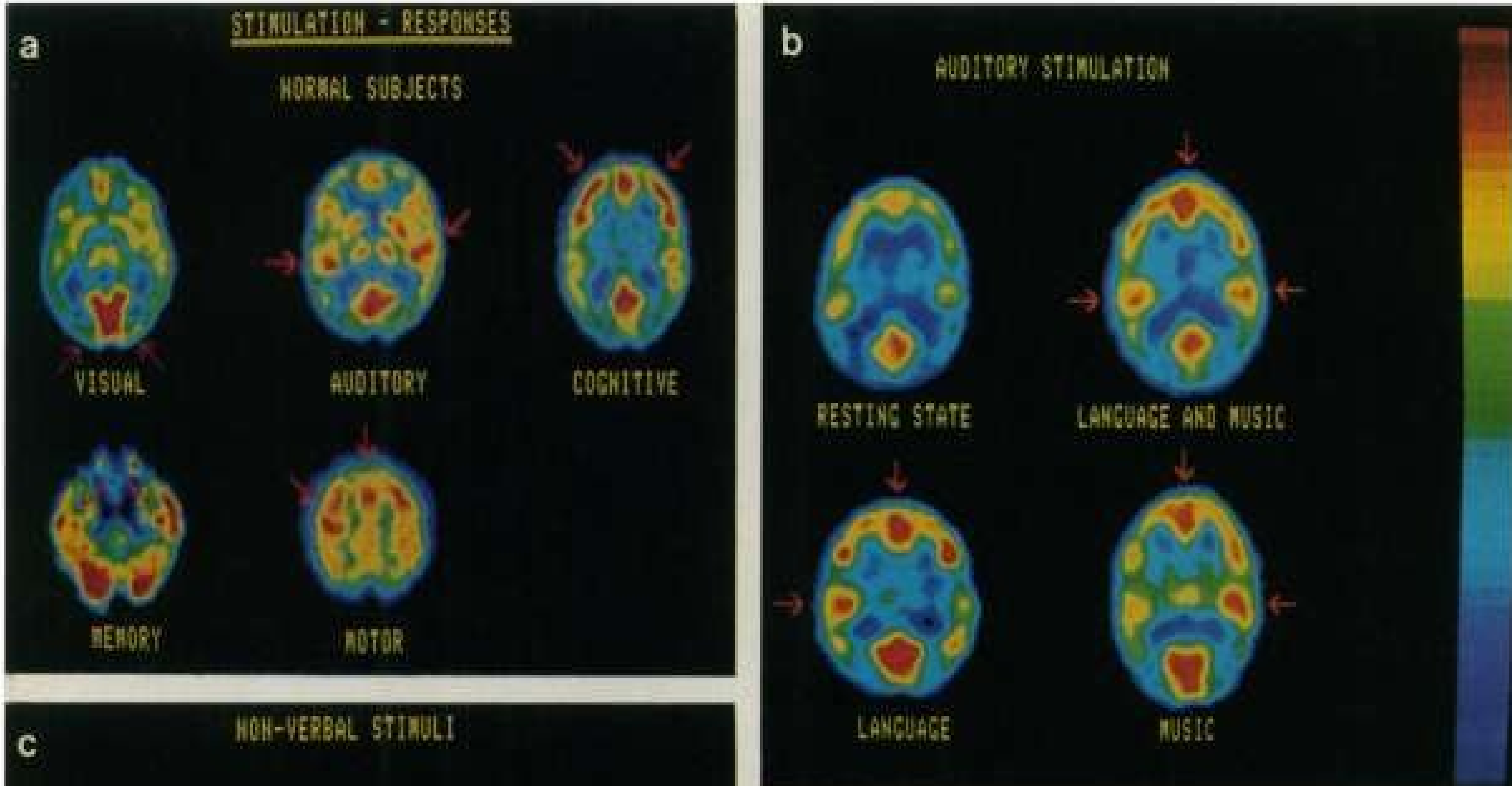
Brains are energy expensive:
brain scaling exponent (slope) = 0.86 > 3/4 for whole body metabolism
(Karbowski, *BMC Biol 2007*).


Homo sapiens sapiens
Homo sapiens neandertalensis
Homo erectus
Homo habilis
Australopithecus africanus
Singe Anthropoïde
capacité crânienne en cm^3
1500 cm^3
1000 cm^3
500 cm^3
Homo sapiens sapiens
Homo sapiens neandertalensis
Homo erectus
Homo habilis
Australopithecus africanus
Lukeino (Baringo)
Ancêtre du millénaire
Singe Anthropoïde
Primate
Préhominidés
Hominidés
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0
-1,7
-0,7
millions d'années

the conventional radiological imaging techniques such as x-ray CT, angiogra-

2a) (*17, 18*). Patients with generalized forms of epilepsy (such as major motor

mals (glucose utilization, blood flow, morphology, electrophysiology, ligand



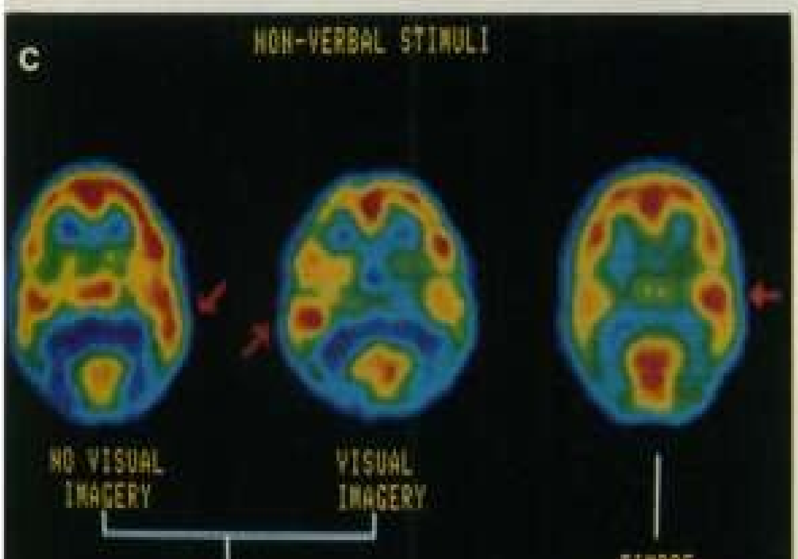
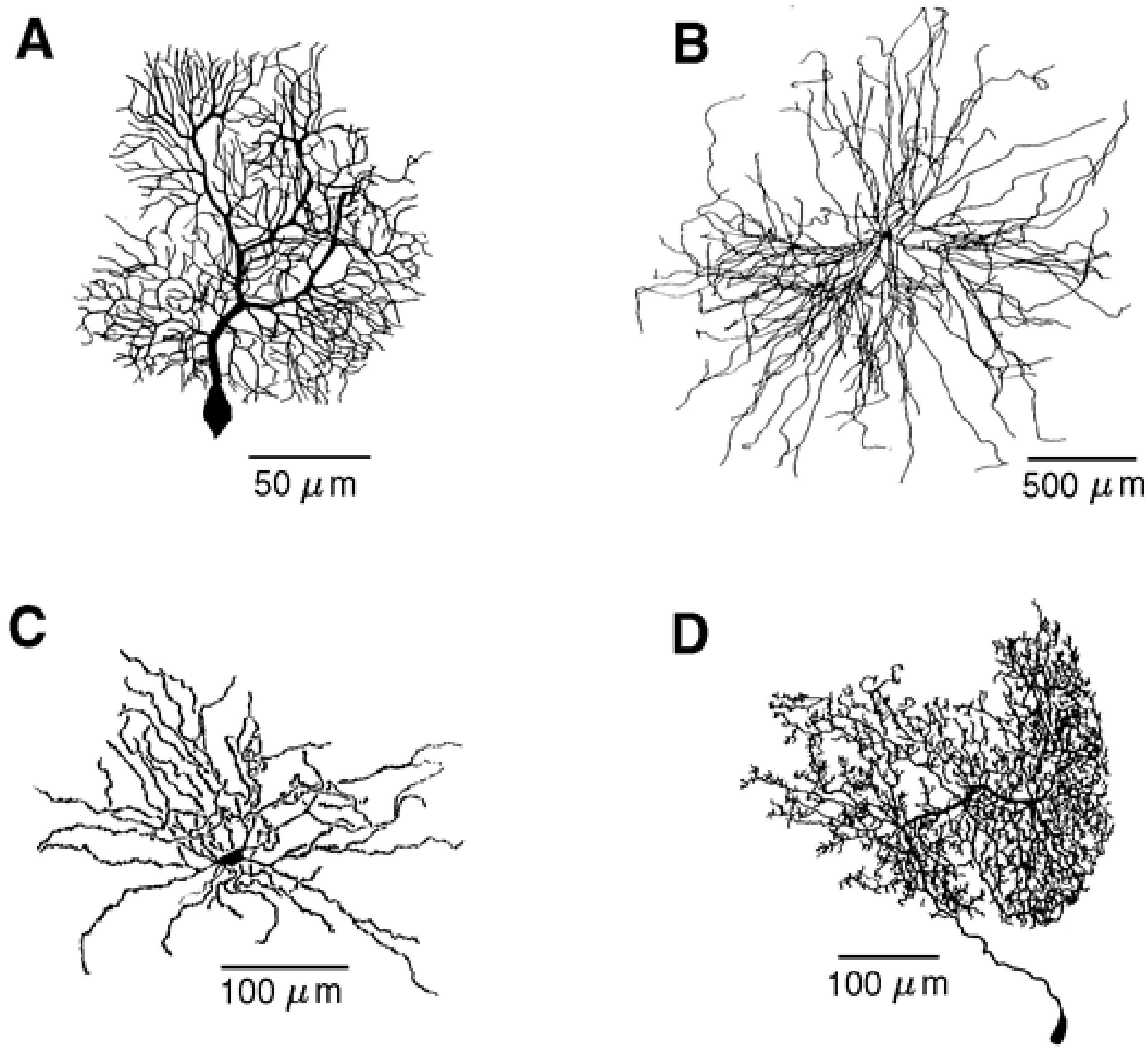
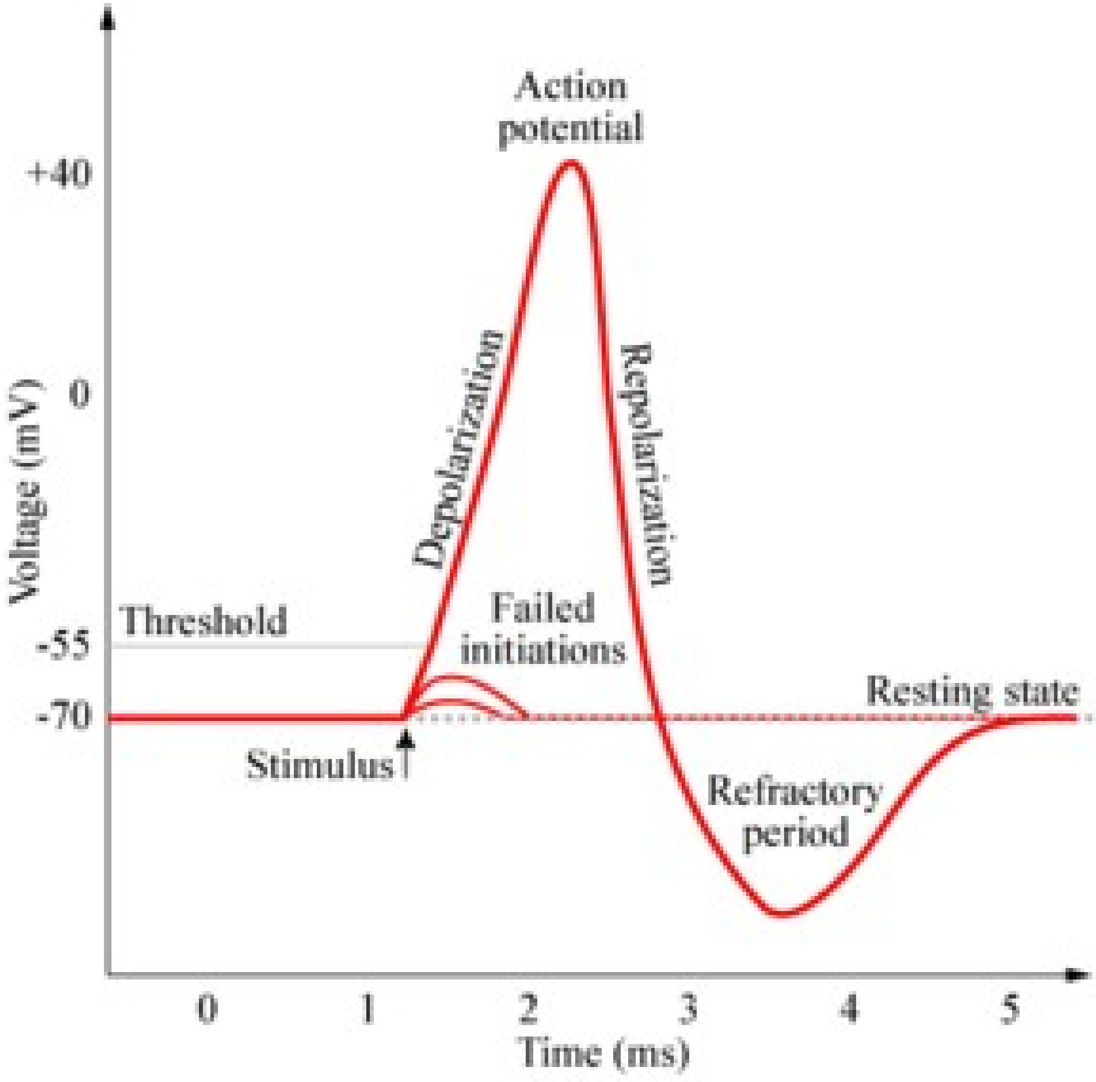


Fig. 4. Functional neuroanatomy demonstrated in FDG-PET studies of normal subjects. (a) Visual stimulus consisting of full-field alternating checker-board stimuli activated (27.6 ± 6.5 percent versus control) the full extent of the primary (medial) visual cortex (arrows). Auditory stimuli resulted in responses (17 to 25 percent increases compared to the control) that were symmetric in magnitude but asymmetric in distribution for the transverse temporal cortex (arrows). These asymmetries in glucose utilization closely match anatomical asymmetries for this area (Heschl's gyri and the planum temporale of temporal lobes). Image of cognitive task represents responses where a specific task was involved rather than passive perception of stimuli; arrows indicate activations [11 ± 3 to 37 ± 15 percent for different types of auditory tasks (29)] of frontal cortex. Memory tasks (remembering information from a verbal auditory


A
50 μm
B
500 μm
C
100 μm
D
100 μm

Neural activation



Information flow through neurons

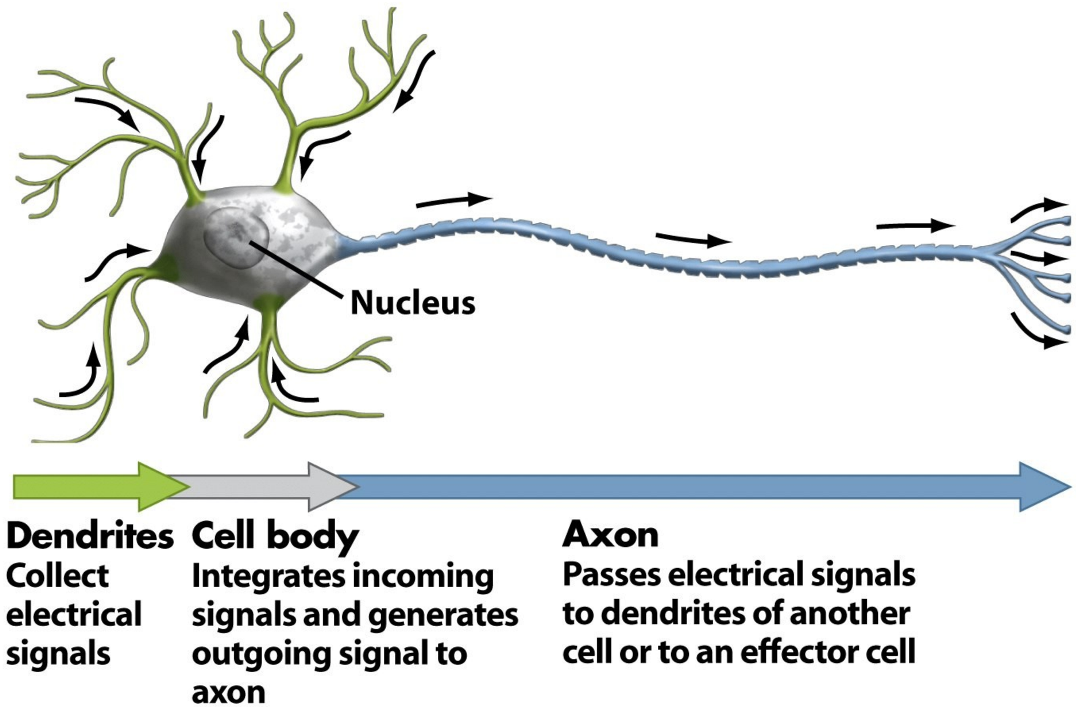
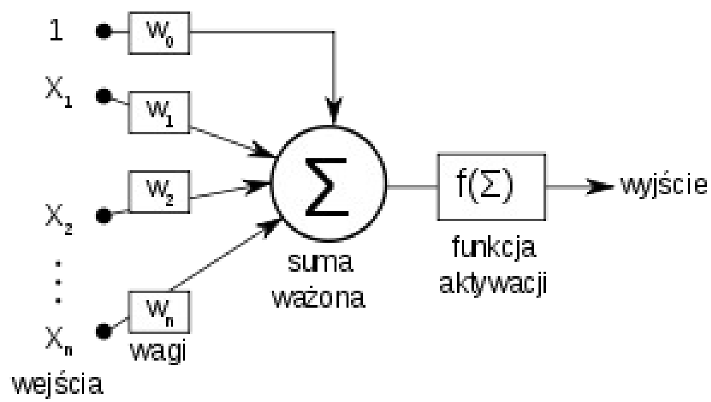
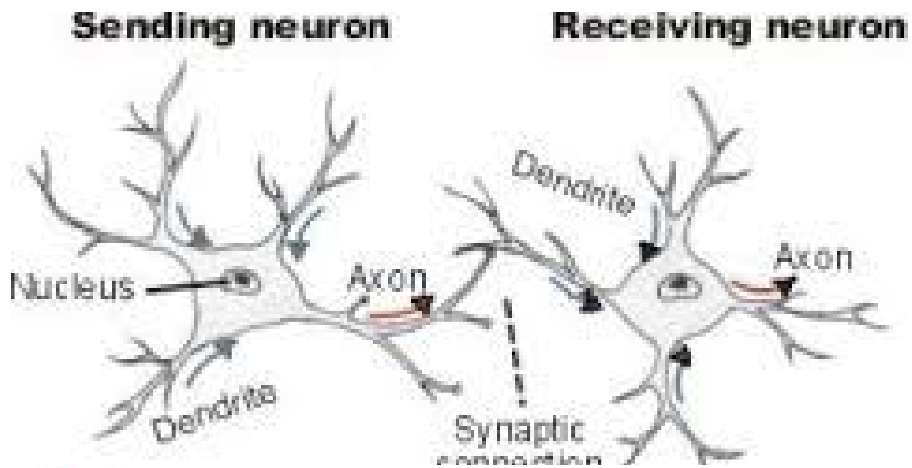
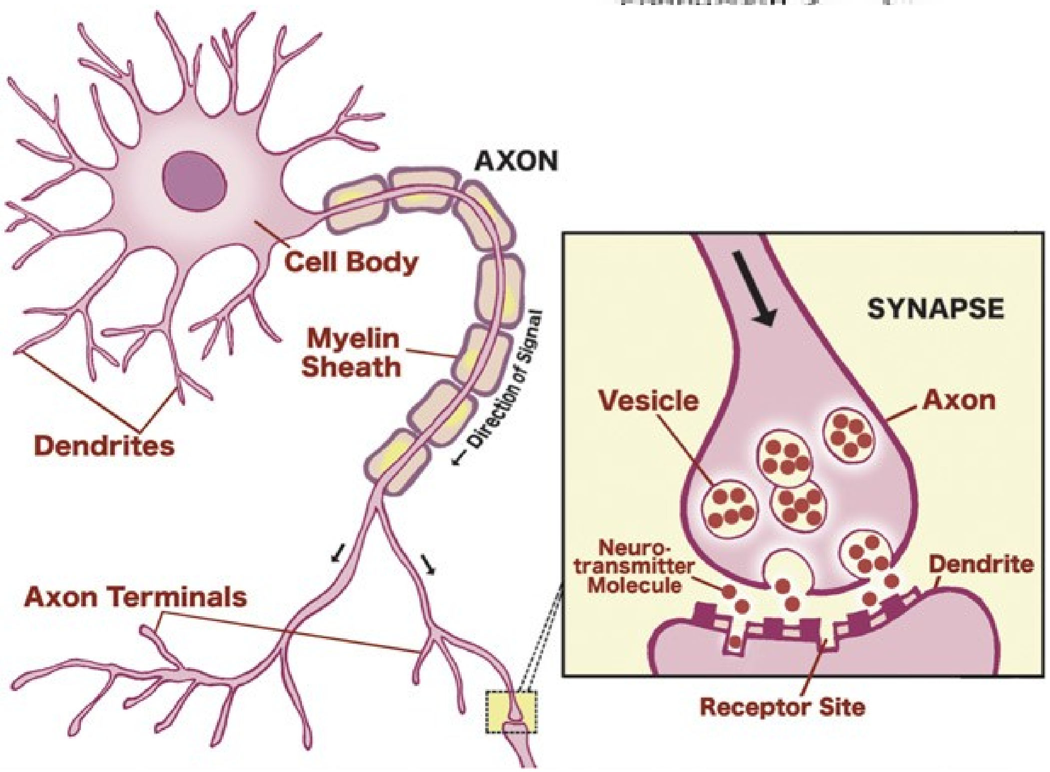


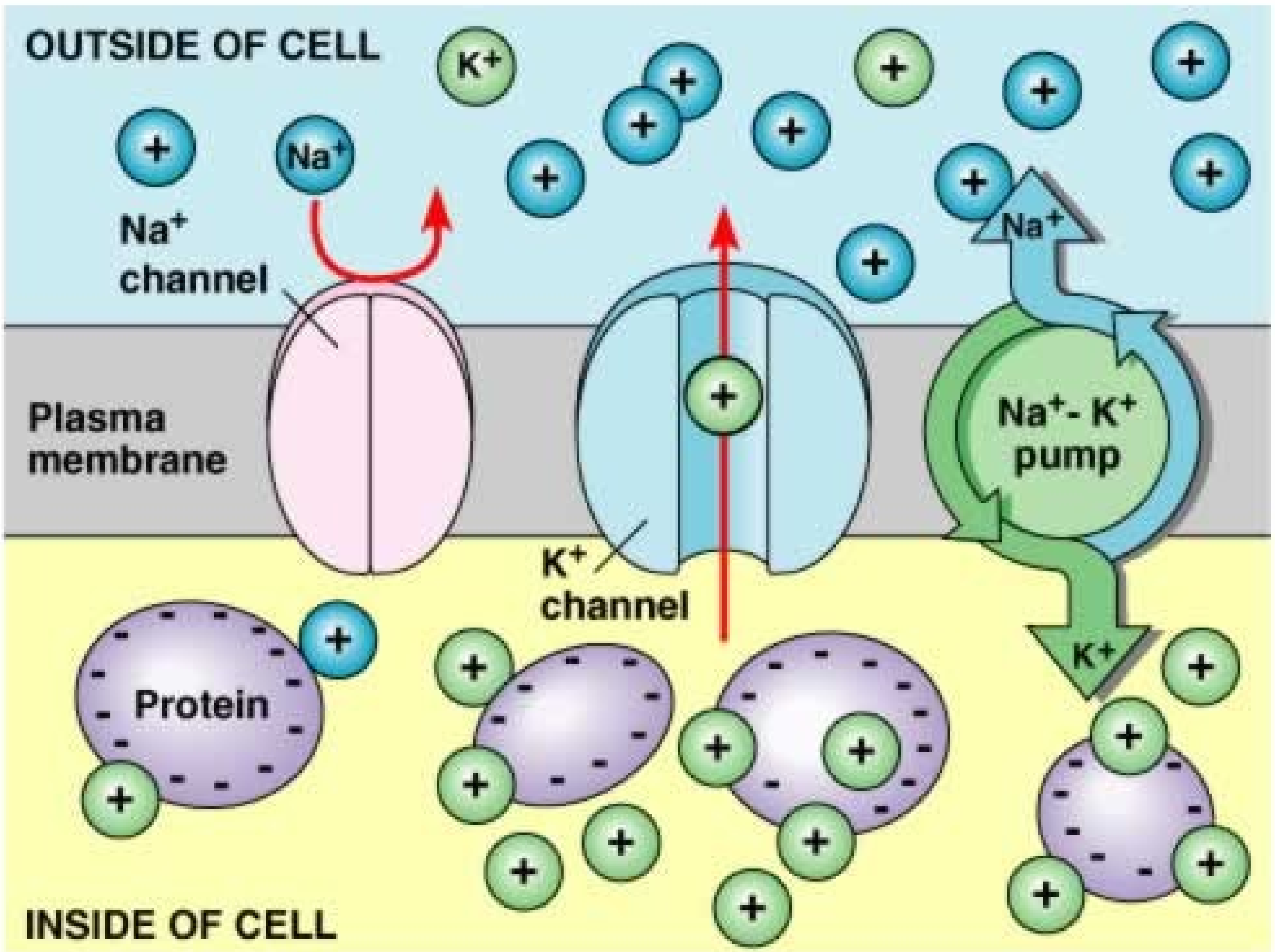
Figure 45-2b Biological Science, 2/e




Sending neuron
Receiving neuron
Dendrite
Nucleus
Axon
Axon
Dendrite
Synaptic


AXON
Cell Body
Myelin Sheath
Direction of Signal
Dendrites
Axon Terminals
SYNAPSE
Vesicle
Axon
Neuro-transmitter Molecule
Dendrite
Receptor Site

Information transfer ≈ ion flows



Voltage and Na-K dynamics

$$CS\, dV/dt = - g_{Na}S(V-V_{Na}) - g_KS(V-V_K) - g_LS(V-V_L) - I_p - I_s$$

$$UF\, d[Na]/dt = - g_{Na}S(V-V_{Na}) - 3I_p - \alpha I_s$$

$$UF\, d[K]/dt = - g_KS(V-V_K) + 2I_p - \beta I_s$$

$$I_p = AS\, [Na]^3/([Na]^3 + \theta^3)$$

$$g_{Na} = g_{Na,max}\, m^3\, h, \qquad g_K = g_{K,max}\, n$$

F – Faraday constant, U – neuron volume

Power generated by Na/K- ATP pumps

$$P_{ATP} = (N/T) \int dt \left(- 3I_p(V-V_{Na}) + 2I_p(V-V_K)\right)$$

N – number of neurons in gray matter

$$P_{ATP} = NSA[Na]_a^3 (3V_{Na} - 2V_K - V_r)/([Na]_a^3 + \theta^3)$$

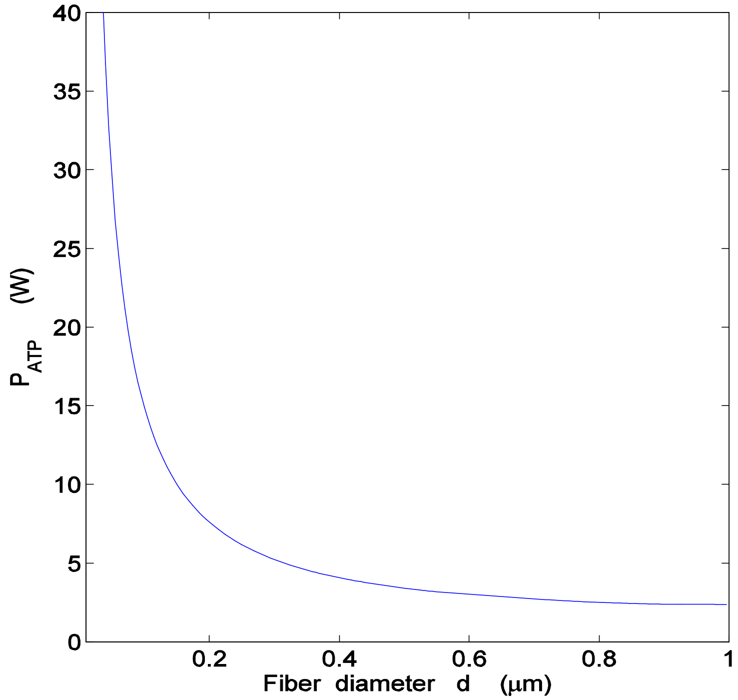
Power generated – result

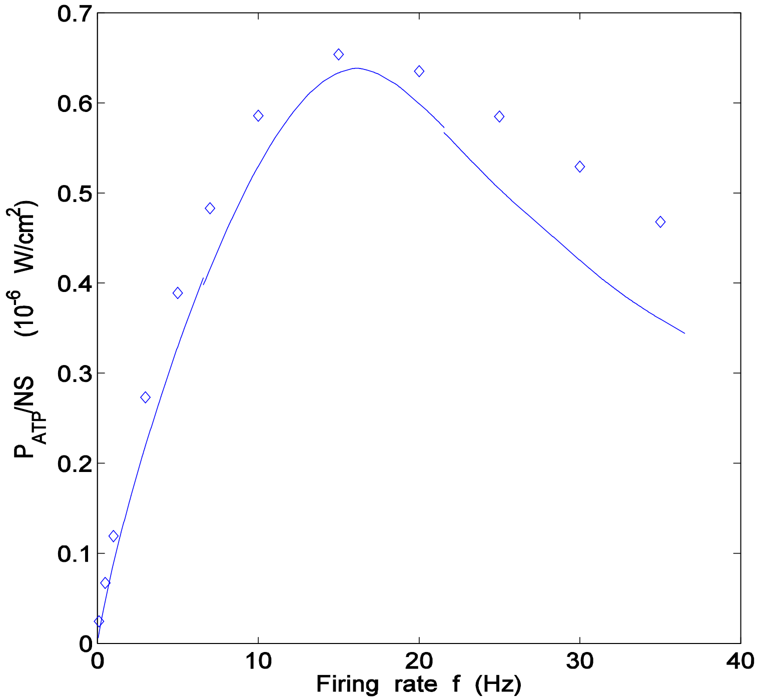
(Karbowski – *J. Comput. Neurosci. 2009)*

$$P_{ATP} \sim (U_g/d)\,(g_{Na,o} + \text{syn.cond.} + f(C+\delta C))$$

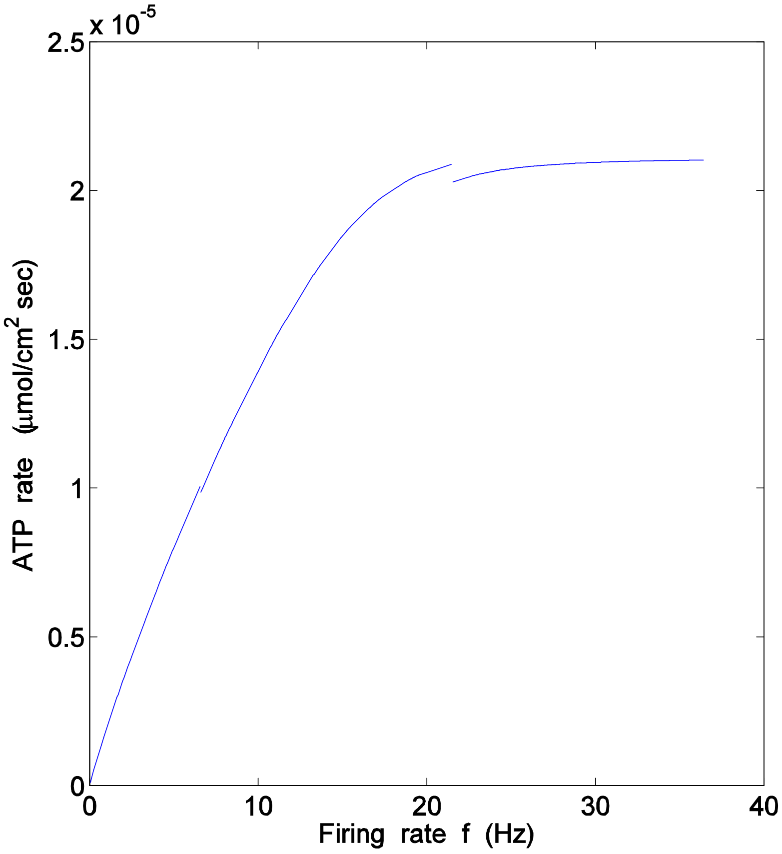
For thin fibers ($d \rightarrow 0$) the power P_{ATP} diverges and much heat is generated!

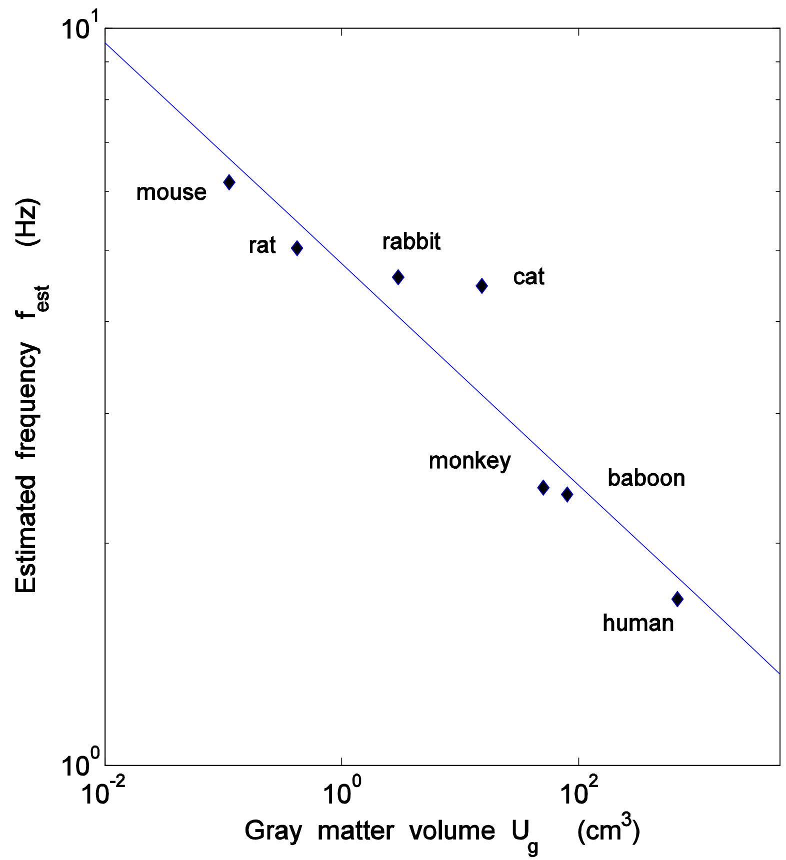
Danger of brain overheating!





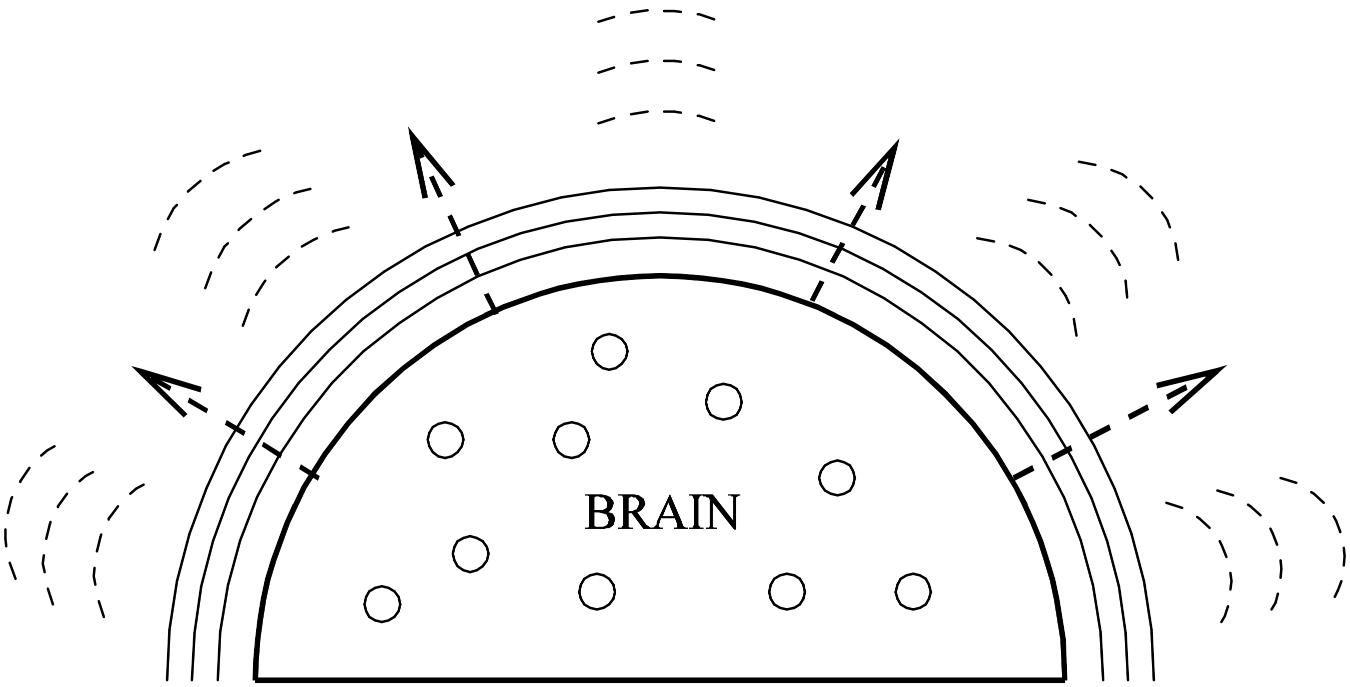
ATP, firing rate, and brain size





Heat removal from the brain

- brain tissue conductance,
- scalp conductance and radiation,
- cerebral blood flow (deep in the brain)



Heat balance equation

$$\rho_{br} c_{br} \, \partial T/\partial t = \kappa \, \partial^2 T/\partial r^2 - \rho_{bl} c_{bl} \, CBF(T-T_{bl}) + P_{ATP}/U_{br}$$

T – brain tissue temperature

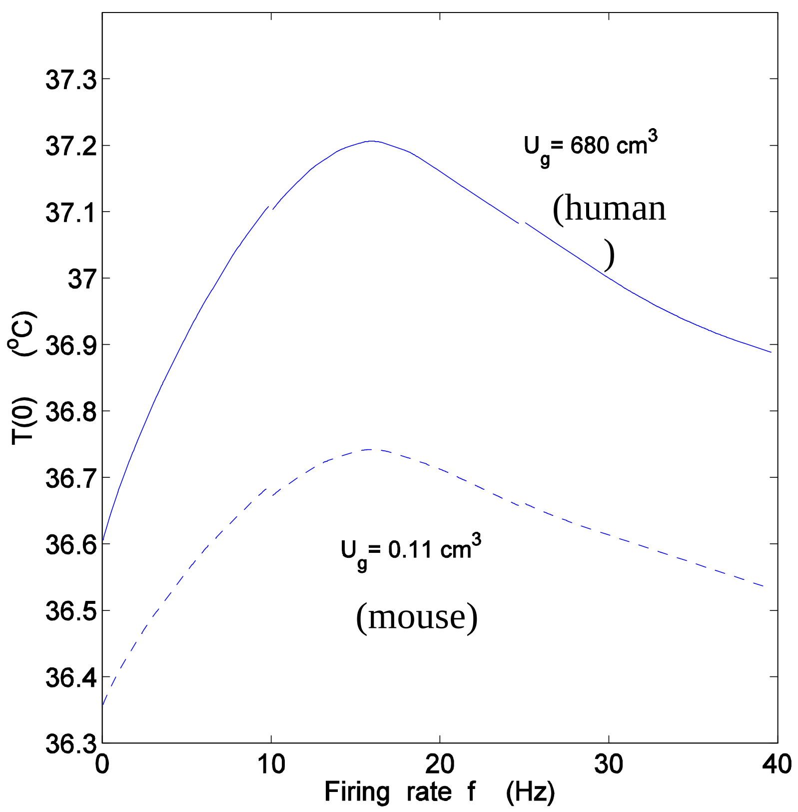
κ – brain thermal conductance

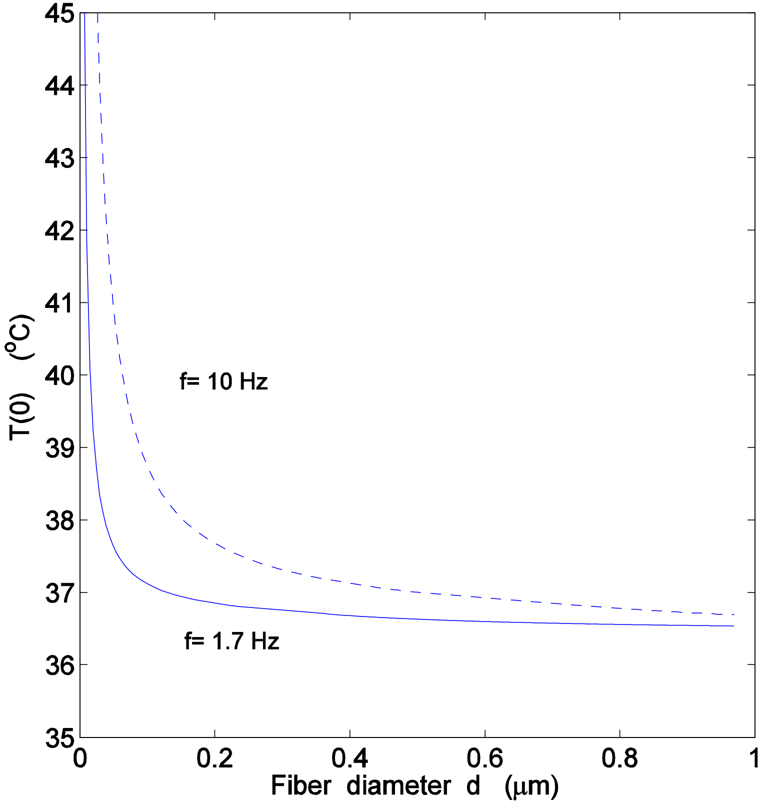
T_{bl} – cerebral blood temperature

r – spatial position

usually $T_{bl} < T$ and thus blood flow removes heat

Dependence of brain temperature on firing rate and fiber diameter





Conclusions

- Brains use more energy than other tissues in the body.
- Thermal properties of mammalian brains depend strongly on firing rates and axon diameter.
- Brains are safe from overheating (too thick axons).